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ABSTRACT

Mathematics teaching is most effective when practice situations cover appropriate material, require constant thought, are transferable to new situations, make sense to the student, and are subject to analysis of components. Teachers must carefully seek the real source of errors and insure that students are as concerned with processes as with results. Levels of competence required of students should be consistent with the anticipated use. In such areas as decimals and derived results from measurements, students may be confused by nomenclature or explanations inconsistent with previous learning. Students must also be taught to apply their mathematics education to real life. Television and film can be used profitably here to present situations designed to be meaningful to a variety of students. Students should be encouraged to think about problems and impose interpretations on them based on their own experiences rather than simply using their minds as storage receptacles for information imposed on them from the outside. (Author/VT)

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Background Paper

Teaching Mathematics: An Integration of Skill and Knowledge

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TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) AND USERS OF THE ERIC SYSTEM."

by
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The Essential Learning Skills Television Project is a cooperative activity of state and provincial agencies, designed to strengthen learning skills instruction in American and Canadian elementary schools. It had its origin in a 1973-74 series of meetings of educators and educational broadcasters in the United States and Canada.

Organized and managed by the Agency for Instructional Television, the project is financed largely by the participating states and provinces. Exxon Corporation is supporting initial informational activities; the Corporation for Public Broadcasting has reserved funds to assist production.

The project undertakes to develop television and related printed materials that will enhance the teaching of communication skills, mathematics skills, and reasoning and study skills. This publication is one in a series of background papers prepared to help in the conceptualization of materials for the project.

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Teaching Mathematics: An Integration of Skill and Knowledge

Abstract

Mathematics teaching is most effective when practice situations cover appropriate material, require constant thought, are transferable to new situations, make sense to the student, and are subject to analysis of components. Teachers must carefully seek the real source of errors and insure that students are as concerned with processes as with results. Levels of competence required of students should be consistent with the anticipated use. In such areas as decimals and derived results from measurements, students may be confused by nomenclature or explanations inconsistent with previous learning. Students must also be taught to apply their mathematics education to real life. Television and film can be valuable here by presenting situations designed to be meaningful to a variety of students. Students should be encouraged to think about problems and impose interpretations on them based on their own experiences rather than simply using their minds as storage receptacles for information imposed on them from the outside.

Introduction

This paper presents some thoughts on the status of mathematics education for the nine-to-twelve-year-olds in grades five or six and considers some possibilities for providing a more substantial foundation for the mathematics education they will acquire in the future.

Beginning in the late 1950's and continuing into the 1970's, the United States and other parts of the world experienced what has come to be called a revolution in mathematics. In fact, the National Council of Teachers of Mathematics not only published a bulletin, *The Revolution in School Mathematics* (1961), but also subsequently published a bulletin called *The Continuing Revolution in Mathematics* (1968).

But are we still in that first revolution of the fifties and sixties, or are we in a different revolution, or are we now in a period of stability? Only the historian will be able to answer such a question. However, those of us concerned with mathematics education have the responsibility of planning for the years 2000 to 2040 when our present fifth- and sixth-grade students will be carrying the major responsibilities for industry, government, and the total social order. The mathematics education they receive today must provide the foundations for the mathematics they will need to function competently in the twenty-first century.

Practice Conditions

The January 1977 issue of *Mathematics Teacher* carries a publishing house advertisement with this heading in bold type: "There is only one way to learn basic Math skills cold. Practice, practice, and more practice" (p. 26). No doubt the company has placed some con-

ditions on their concept of practice as they want the users of their text to carry it out. They surely realize that while practice is an element necessary to the development of skills, it is not sufficient. What, though, are the conditions publishers should build into practice activities if students are to retain skills for long periods of time even without using them? Teachers of fifth and sixth graders are preparing their students for the twenty-first century; what a ten-year-old can do today is important only insofar as it provides for his future needs.

Teaching must be done so that the learner does more than store information for future retrieval. The learner must be selective, must make current interpretations, and must relate different ideas and concepts for fuller meaning and better retention. One can only do this through continually thinking about processes, raising questions, making comparisons that help build new ideas, and noting how mathematics and the environment are integrated.

K. Patricia Cross (1975), in an address at Iowa City, made the following comments:

> The human mind is a marvelous thing, but we abuse it when we assume that storage and retrieval of information is its greatest asset. In our rapidly changing world there is no permanent body of knowledge that can be passed on to our youth in the hopes that it will last a lifetime. There are only intellectual skills and attitudes and appreciations about learning that will help people use their minds throughout their lifetime.

The first condition for practice activities is that they be drawn from the appropriate domain. What are the priorities for judging what to practice to proficiency and what to delete? Times change and so do needs. Not too long ago, by completion of elementary school all children were expected to recall immediately each multiplication fact through the twelves, to recall all the perfect squares through twenty-five, and to use an algorithm to find the square root and cube root of any number. Is there any need to meet this standard today? What will be an appropriate standard of learning for the twenty-first century? The student cannot make these decisions, and while the teacher can only make judgments about them, these judgments are very important and must be made.

The second condition for effective practice activities is that the thinking process must accompany them. When you reach the point where the act becomes instinct or habit, you no longer improve because there is no involvement. Each act must include a thought process if practice is to be effective.

The third condition is that students be able to transfer the skill established through practice to new situations. For example, we often find students unable to add two-digit addends, such as 49 where

exchanging tens and units are necessary. This difficulty can often be traced to developing a skill that was satisfactory in two-digit addends where no exchange was necessary. In the exercise $\begin{array}{r} 42 \\ +\ 25 \end{array}$, the student may add from left to right. He always gets a correct answer, and the more firmly he establishes the left to right habit, the more difficulty he has when conditions are such that left to right addition complicates the situation.

The fourth condition is related to the third in that the environment of the practice and the conditions under which it is carried out must make sense to the students. If students always get the desired results by adding from left to right, it makes little sense to them to change. The only alternative left to the teacher is to make sure the students' original efforts do not result in practices that must be changed in the near future.

The fifth condition is the most complex and the least likely to be efficiently met. This condition is that there be a complete analysis of the elements of the practice situation. One needs to be able to recognize each of the component parts of the activity but at the same time treat these parts as elements of the whole and not as isolated entities. It is not at all uncommon for students to recall with ease the necessary addition facts, but when confronted with the task of using the facts in a series with one following the previous one through the series, they are unable to function effectively. The students' errors may look like weaknesses in the knowledge of basic facts, but if we attempt to correct their difficulties at that level, we are likely to fail.

Source of Errors

The source of errors observed by the teachers must be ferreted out. This source is seldom obvious, but it is at the point where the error originates that efforts in correction must take place. When working with decimals, for example, $.3 \times .2 = .6$ is a common error. If we attempt to treat this difficulty as a decimal difficulty, we are again likely to fail because the real problem facing the student is related to the concept of fractions. In the fifth and sixth grades, we do put considerable emphasis on decimals and percents, but students often have difficulty because we do not identify the true nature of the symbolic representation. Decimals and percents must be treated for what they are—a subset of the fractions. The decimal point and percent sign are other ways of representing fractions. If the student makes the error $3/10 \times 2/10 = 6/10$, then, of course, the culprit is multiplication of fractions. The problem faced by the teacher is determining at what stage the difficulty originates. There is no future in attempting to cure the difficulty by working on the symptoms.

Active participation is needed so that the students understand why interpretations vary and why different processes may require different levels of skill. Then they will also be able to create new concepts and language, and pose new problems.

Activities designed to increase a student's competence and security through practice often do the opposite because we sometimes say one thing and mean something entirely different. For example, we talk about baseball batting averages of three hundred twenty-eight percent when we mean thirty-two and eight tenths percent. Elementary school students need help and support in straightening out these commonly accepted errors. This is not all bad. The world is not perfect, and it is not our task to change widely accepted conventions. But what we must do is confront students with these conditions so that they can put them in proper perspective. The student who has no trouble with .328 as being 32.8% will have no trouble when someone names it something else. Changing nomenclature is a way of life, and while one knows that the family car is sometimes called a Ford, Mustang, or junk heap, one must first understand the term car, just as the student must first understand that 328/1000 is a fraction, regardless of how it is written.

Since the very early research on how children learn and what learning strategies obtain the best results, we have known that overlearning is necessary if one wants to retain information to retrieve on demand. However, as Ms. Cross pointed out, there is a better use for the mind than storage and retrieval of all important information. There was a time, no doubt, when the only storage space we had available was the human mind. But today there are many readily accessible places where knowledge can be stored. The entire Educational Resources Information Center (ERIC) is a good illustration. Most libraries have computer systems that will locate sources of information when the system is asked the right question. One function of the teaching process is to distinguish between knowledge that one must store in the mind for use either directly or in a reconstructed form and knowledge that one can store in the other facilities. We need to remember that the decisions being made today will affect our functioning in the twenty-first century. The student must be a part of this decision-making process, must be involved in the thinking that takes place. If the student is asked to do single addends addition until the result is correct ninety percent of the time, will the level of competence be greater or less than is needed? The use a student will make of the skill in a large measure determines the answer to that question.

Process vs. Answers

The concept that different levels of competence are suitable for different uses is one we often overlook in school. If we see an incorrect solution, we often simply say it is wrong and must be done again. We do not give the student enough credit for having done many things correctly, even if the result didn't come out as expected. To draw an analogy, if one travels from New York to Los Angeles and makes a wrong turn on the interstate at Denver, one doesn't go all the way back to New York and start over. Why shouldn't we help students to find where they went wrong and correct the error there? The concept that the proof of the process is a satisfactory product is really

a myth in most instances. If the doctor gives you a series of pills and you get well, he is hardly in a position to cure you a second time unless he knows in what sequence he gave the pills, the chemical nature of the pills, how those pills react with each other, and how they react with your body chemistry. Only with all this information is his process a valid one. Nothing is wrong with accidental solutions. In fact, some of the greatest discoveries in the world were accidental. But the process that led to the discovery, and not the result itself, was the focal point of the subsequent action. When an event happens that you want to happen again, the immediate task is to determine what variables there are in the situation, how the variables performed, and how one can control the variables to assure the repetition of the event. Therefore, the important thing is not the accident itself but the action that the learner takes to capitalize on the accident.

In the January 1977 *Arithmetic Teacher* there is a good illustration of how a student who stumbled on a process yielding correct results was able, with the help of his teacher, to use his observation to establish a general method. Nick was changing repeating decimals to fractions, but he was also involved in what he was doing and was carefully observing the process. He observed that, "We simply subtract the non-repeating portion of the decimal from the entire repeating decimal. This gives the numerator of the fraction. . . . Use one nine in the denominator for each digit that is repeating. Since the denominator must have the same number of decimal places as the original problem, fill in the remaining places with zeros" (p. 81). Nick tried out his system on several problems and drew up the statements that described the process. From here he solved some more general problems. This doesn't prove that there is not a repeating decimal where the process doesn't work, but it does show why learners must always be involved—by thinking, observing, questioning, and saying "what if" as they continue to develop their knowledge and increase their skill in its use.

The mind of man is more than an electronic computer. True, we spend some time collecting, storing, and retrieving data, but we do more than that. We have the capacity to think, to see beyond the pattern that activates the circuit, and to recognize a significant element in spite of its disguises. We do more than rearrange discrete elements into new patterns and new designs. We can make adjustments to compensate for not receiving the entire set of directions; we can also adjust by filtering out extraneous data and ignoring false logic that tends to misdirect our attention.

Teaching must provide the learner time, opportunities, and motivation to think, make interpretations, seek alternatives, ask questions, and pose new problems if he or she is to develop competence for the year 2000. As teachers, we often forget that our minds are carrying out these activities. Therefore, we tend to bypass teaching techniques that will help the learner develop these discriminating abil-

ities so that at maturity his or her mind will function in the same way. This is not an endorsement of the faculty psychology position—that use alone sharpens the mind. Our mind does not function better just by use any more than does our car, stove, or refrigerator, but these appliances can be made to operate more efficiently by continual observation and analysis, and by appropriate adjustments. In the teaching situation, the students have the opportunity to let their minds go beyond the immediate difficulty to other potential difficulties stemming from the same problem. Thus students develop the optimistic philosophy that somewhere along the way there is a resolution or solution to the situation. Regular classroom work seems to be one of the few places in life where it is appropriate to give up before you start. In arithmetic, fifth and sixth graders need experiences in which they cannot solve the problems but can still live with the situation. Maybe you can't fix a leak in the roof, but you can catch the drip in a bucket. A student can't get an answer but learns why not; this is a far better education than getting the answer without knowing why.

Confusing Presentations

Consider the following incident that recently occurred in a fifth-grade classroom. To add to the meaning of the place value and at the same time teach decimals and the monetary system, the teacher and students did as follows:

Teacher:	How would I write a numeral to show that I had 823 pennies?
Joe writes:	823
Teacher:	How would I write a numeral to show that I had eight dollars and twenty-three cents?
Joe writes:	$8.23
Teacher:	How would I write a numeral to show that I had twenty-three cents?
Joe writes:	23

Nancy had been puzzled and timidly asked, "Didn't Joe write eight and twenty-three hundredths dollars?" The teacher, knowing the mathematics should be relevant and related to life, went to her purse and brought out a cash register tape from a grocery store, explaining to the students what she had and asking if they had ever seen such a tape. All had. The teacher gave it to Bill, who read:

Meat	2 dollars and 94 cents
Groc	3 dollars and 16 cents
Total	6 dollars and 10 cents
Out of	10 dollars
Change	3 dollars and 90 cents

Sam was then asked to copy for all to see what was on the tape:

MT—$ 2.94
GR— 3.16
ST— 6.10
CA— 10.00
CH— 3.90

Teacher: Is it now clear, Nancy, that what Joe wrote does represent eight dollars and twenty-three cents?

Nancy, still frowning, did not reply and the class continued to practice writing $2.85, $ .97, $8.64 until each had the skill.

Probably only Nancy was aware that they had not touched the crux of the problem. There are few absolutes even in arithmetic. How many equivalent representatives are there for $8.23? One is 8 dollars, 2 dimes, and 3 cents. If you want to make a phone call from the desert, this is a very important representation. Others are: 823 pennies; 8 dollars, 4 nickels, and 3 pennies; 16 half dollars, 2 dimes, and 3 cents; 8 half dollars, 16 quarters, 2 dimes, and 3 cents; and there are more. One that you cannot do is 8 and 23/100 dollar bills. But, don't miss the fact that our system of writing says just exactly 8 23/100 dollars, just as we use 7 1/2 apples when we have an apple cut in half.

Since the school is a subset of society, concepts existing in society are not always evident in the school. Teaching in school must be such that the learner can adapt his or her concept to deal with the larger society. One of the errors we made in the mathematics revolution was to assume we could reform all those layman who were not using our language properly. Finally we realized that language is a part of the society and grows with it. We would be as well-advised to teach our mathematics students the language of the world and how it translates into the language of mathematics as to teach the opposite. In teaching-learning situations, we must keep the duality of language constantly in mind. As language users, teachers must always be conscious of their need to help students make transitions between words of multiple meanings, words that we accept in common usage even though that usage is not compatible with strict mathematical usage. Let us go back to the example of $8.23. We would not accept 8.23 ft. as anything but eight and twenty-three hundredths feet. If it were 8.23 meters, we would probably read it as 8 23/100 meters, not 8 meters, 2 decimeters, 3 centimeters.

Derived results from measurements are a glaring example of saying that we are doing something that is in fact impossible, but saying it anyway because the answer comes out correctly. To find the area of a rectangle, we use the formula $A = LW$, which says the area of a rectangle is the product of the length and the width. For the volume of a rectangular solid, we use $V = LWH$, which says the volume is the product of the length, width, and height. The distance a freely falling body has covered, is $S = 1/2gt^2$, where S is distance, g is gravitation, and t is time: The formulas need not be changed; after all, they are handy to have if you can recall them. The difficulty is that the student who really understands the nature of multiplication knows that the way we say we use the formula is not possible without redefining multiplication. It is curious to note that when we had little concern for mathematics as it related to real life, many of the current

problems did not surface. Many concepts are easily accepted when considered abstractly, but when that abstract concept is translated into some real situation, such as the computation of the area of a rectangle 5′ by 3′ the concept must take on some added dimensions in the formula $A = LW$ and come up with the area as 15 square feet. Our thoughtful student recalls the nature of multiplication as a shortened process for doing addition. This makes the student wonder how 5′ and 3′ are used in an addition concept to arrive at 15 with a designation of square feet. Recalling that 2 x 6 represents the union of 2 sets of 6 elements each, the student immediately sees how one could take 3 and 5′ as three sets of five foot lengths, but has no concept as what should be done with 3′ x 5′. Three feet represents a length and so does five feet; neither, therefore, can be used a multiplier. *Multiplier* is an action word telling the user how many times to do something; it is not the description of a concrete object. We need to help the students see what we are really doing and not try to confuse them by saying we are doing something different. They need to know the unit of a measure being used for area is a square—in this case, a square foot. They need to know we are attempting to find how many of these squares we will need to cover the region in question. When they know the above, they can apply the formula without violating what they have already learned. They see that there are 5 squares in 1 row and they have 3 of these rows. The problem now meets their conditions—3 x 5 rows of squares. That means 3 sets of 5 squares, which provides the 15 squares needed. If students fail to find the area of the rectangle, we must determine if it is because they can't handle the mathematics, or because we have changed the rules without telling them. No teacher would do this intentionally, but we are all creatures of habit. We do things without thought; we have ceased to be involved in our own activities, and, therefore, are no longer in control of them.

There is a place for pure abstract mathematics, completely unrelated to the existing world, but it is not the fifth and sixth grades. Mathematics must be in a sensible environment for the students, who must take time to think about the total situation as well as its subelements before putting their pencils to the paper.

Definitions, rules, algorithms, and facts are all necessary elements in the teaching-learning process, but they are not sufficient. The teaching of mathematics must be very dynamic. The problems in most arithmetic books have been stripped of all their excitement; they are so sterile the student has as much fun working them as an adult has eating baby food. They don't look sterile to adults because they may unconsciously place the problem in settings of their experience and relive an exciting time. The student with only limited experiences will rarely find a problem that recalls anything interesting.

During the mathematics revolution some text writers tried to write so that more students could identify with the activities required of them. For example, they were asking students to suppose that the

bus wouldn't start, or that it got behind a slow semitrailer. The textbook took on many pages of words. The teachers and students couldn't find the numbers easily, and from force of habit they wanted to get their hands on them immediately and begin their manipulations. The words began to disappear from the text. The authors began to use short cuts without teaching the concepts needed to understand the short cuts. Over the years there has been considerable research to show that the teachers who get the best results are those who spend more time on in-class development of concepts, on meaningful exchange of ideas and questions between students, and on informal investigation in areas where differences occur. The students have greater retention, learning is more meaningful, there is an increased ability to get results independently, and the students are more likely to bring the appropriate knowledge to bear on strange and unfamiliar situations. See, for example, Shuster and Pigge (1965), Zahn (1966), Hopkins (1966), Shipp and Deer (1960), and Davison and Ruddell (1955).

Use of Television

Of course discussion takes time. But if the above research is valid, the total time expended by the learner may actually be reduced by discussion. In the preceding illustrations much time was wasted by lack of consistency in the presentation. We have no way of knowing how much time the students spend trying to decode the puzzle we have inadvertently presented to them. Now with television and film we can present a single situation designed to be meaningful to a variety of students. True, not all students will find the same meaning, but mathematics as a discipline is adaptable to all types of real-life situations.

To illustrate, suppose we are teaching percent and the scene is a department store with various goods of interest to fifth and sixth graders. The goods are all marked down for quick sale at various percentage discounts. The conversation centers around a number of different things, but no mathematics or sales. The viewers all this time are thinking what they would like to buy. So why not have the narrator ask them? Each student selects his or her own problem. Even if two people choose the same object, they probably do it for different reasons. Like all good shoppers, they can compare deals and, in the process, do a lot of mathematics. This alone is not a script for a television presentation because there are many other considerations—how much needed knowledge do the students bring to the class, how much time do students efficiently use in discussion, and what difficulties is it wise for the student to encounter so that the end result will be more meaningful?

Many things occur while the students are watching if they can identify with the material being presented. Not only are they in the process of making a decision about what they will exchange their money for, but they are also thinking about many other elements not on the screen. They are making estimations of costs, doing

mental arithmetic as the result of these estimations, and translating the symbols of numbers, percents, labels, and words into a language meaningful to them, and they are making judgments based on their parameters, not those of some stranger. It may not look like the ordinary classroom situation, but they are solving problems.

Individual Interpretations

The crucial question is the use of dialogue that involves the student in the appropriate thought processes. The situation must stimulate discussion, either real or imaginary, and it must help the viewers seek out and accept alternatives to their first thoughts about the situation. The mathematics of the situation must be reasonable to the viewers. In the area of language arts, the students are asked to read some author's work. The teacher and the students then explore alternative meanings by discussing such questions as: What was the attitude of the main character? What did the author mean by . . .? How would the story have ended if he had let Joe die? Mathematics writing is also a story, and need not be restricted to the commonly known story problems. The teaching process must be such that the learner feels free to make the interpretations needed to round out incomplete presentations.

Suppose a textbook requires students to work the following division exercise: 387 ÷ 29. What kind of a problem does the author have in mind? Is the 387 a pile of pennies to redistribute into 29 separate, equal piles, or is it 387 feet of rope to cut into 29 short pieces, or could it be 387 candy bars to be divided equally among 29 boys and girls. Nobody positively knows what the author had in mind, and it probably is not important. What is important is what the present students have in mind as they work the exercise. All students need not interpret the exercise in the same manner. If it is pennies, the student puts 13 pennies in each of 29 piles and has 10 left that cannot be redistributed. If the student chooses the rope situation, the result is 29 short sections of 13 10/29 units each. The basic arithmetic is the same in all three interpretations, but each student is thinking not only about the necessary computations, but also about the real results the computations provide. If this same question appeared on a test to be machine scored, and the machine decided to interpret the problem as piles of pennies, then only those students who made the same interpretation would be marked correct.

Difficulty of Assessment

In 1972-73, the Education Commission of the States, in a project entitled National Assessment of Educational Progress (NAEP), conducted a nationwide assessment of mathematical competence of people ages nine, thirteen, and seventeen and of adults. The results were not entirely satisfactory, but we would hope to do better in the future. And although we do not know exactly what questions were asked, since only a small sample was made public, we cannot attribute all of the student failure to weakness in basic computation. We don't know what experience the students brought to the problem. Therefore, we can't correctly judge their interpretations of it.

This is the point where discussion must take place. Here the student says, "I assume this problem situation to be *so* and *so* and will solve it in this manner." For example, NAEP found that a number of seventeen-year-olds couldn't balance a checkbook, that only thirty-three percent of the thirteen-year-olds could correctly find the time it takes to drive 275 miles if the average speed is 50 miles an hour, and that only thirty-one percent of the nine-year-olds could estimate the sum of three three-digit numbers to the nearest 100. These are facts and there is no reason to question them, but we need to pinpoint the difficulty. In the checkbook balance, could the difficulty be the place where each item is recorded, or could it be the confusion discussed earlier over the way we represent money? Was the difficulty in determining average speed a division computation problem, or was it a different interpretation of the meaning of average, or might it have been a real life interpretation of average speed as regulated by outside force? Who can tell which, if any, was the problem? Was the approximation difficulty due to incorrect computation or to interpretation of the meaning of approximation? The percent of students getting the result selected as correct by the testers is about what chance selection would give. We cannot improve competence until we know which of the necessary elements need work. We cannot pinpoint difficulties unless there is discussion between teachers and students about mathematics situations as well as about computation processes and their results.

Summary

The position the previous comments present and illustrate is basically as follows:

1. The mind is too important an instrument to spend most of its time storing information. We must carefully select only that which is most useful.
2. We save time by always developing ideas in such a manner that they will complement other ideas rather than detract from them.
3. The mathematics environment and the process must be sensible in the eyes of the learner.
4. The learner must be an active participant, not only in doing the mathematics, but in determining the meaning of the problems.
5. The adult's world is not a child's world. The differences in language and interpretations between these two must always be recognized in the teacher-student relationship.
6. Answers are of no more value than the processes used to get them. Students need to be encouraged and given compensation for clear processes as well as correct answers.
7. In any mathematical situation the learner should be encouraged to make interpretations, to raise "what if" questions, to seek alternative processes for the solutions, to experiment with mathematical ideas, and to pose new problems.
8. We need to promote independence among students in problem solving. Therefore, we need problem environments that will at-

tract each member of the class—this means the same mathematics being used in various situations at the same time.

9. It is vital that considerable activity take place before the pencil is picked up if the work is to be meaningful to the student. Who would read the road map after getting out on the interstate going 55 miles an hour? Students need time and opportunity to think about, talk about, raise questions about, and plan for action before they begin putting numbers on the paper.

All this may seem overwhelming, complex, and time consuming. It really is not. We are simply asking to give students time to think about mathematics and become personally involved. The thinking students will not accept things they do not understand; they will ask for clarification or seek it themselves. Therefore, they are less likely to become confused. With thoughtful practice, they will need less time and with personal involvement, they will work more efficiently. Designing school programs that meet these standards will pay dividends and better prepare us for the twenty-first century.

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Other papers available:
Coping in the 1980's: Inquiry Skills and Participation in Decision-Making for Fifth and Sixth Graders, by Byron G. Massialas, Professor of Education, Florida State University.

Communication, Fifth and Sixth Graders, and The Essential Learning Skills Television Project, by Mary Lou Ray, Language Arts Coordinator, Mississippi Educational Television Authority.

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